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Parallel Simulation of Blood Flows in 3D Patient-specific Arteries

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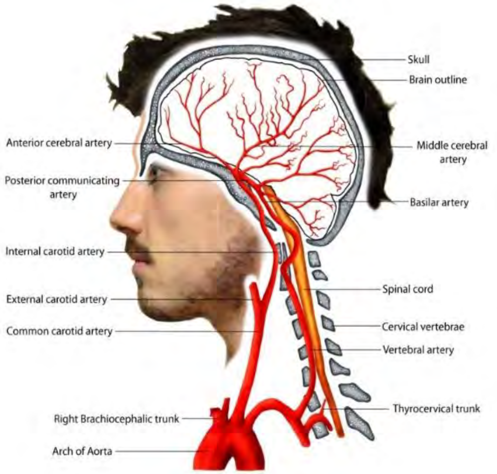
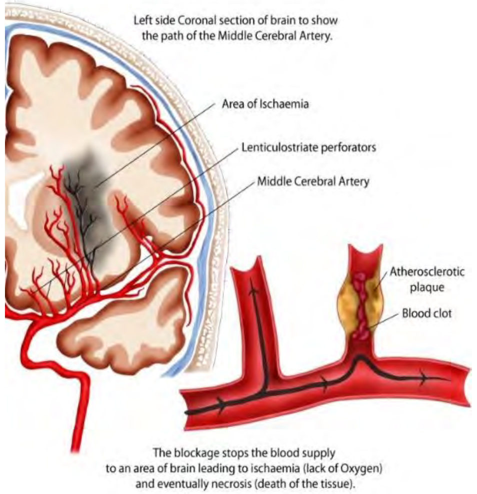
Motivation

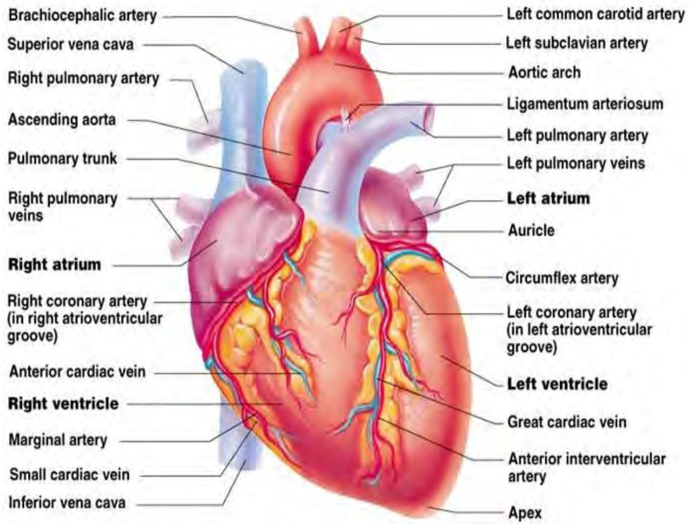
The cerebrovascular system





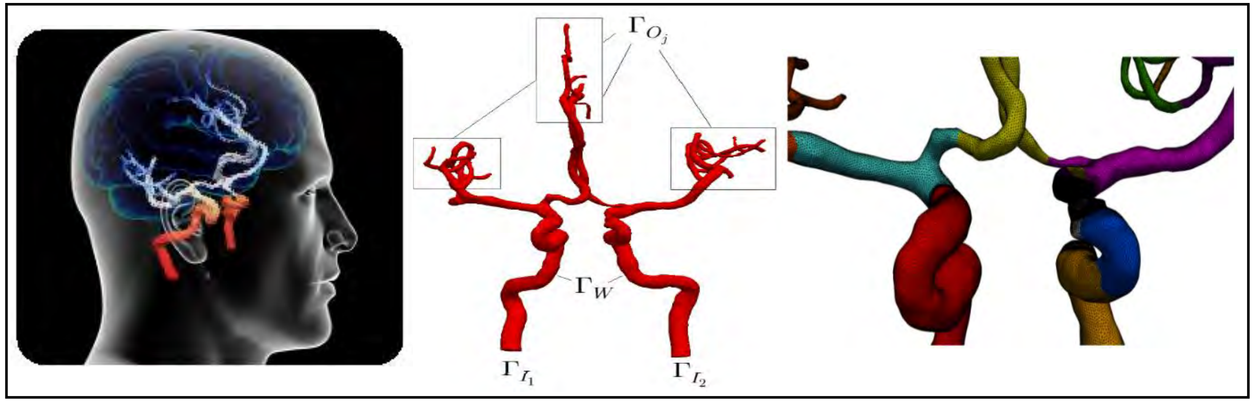
https://neuro4students.wordpress.com/pathophysiology/

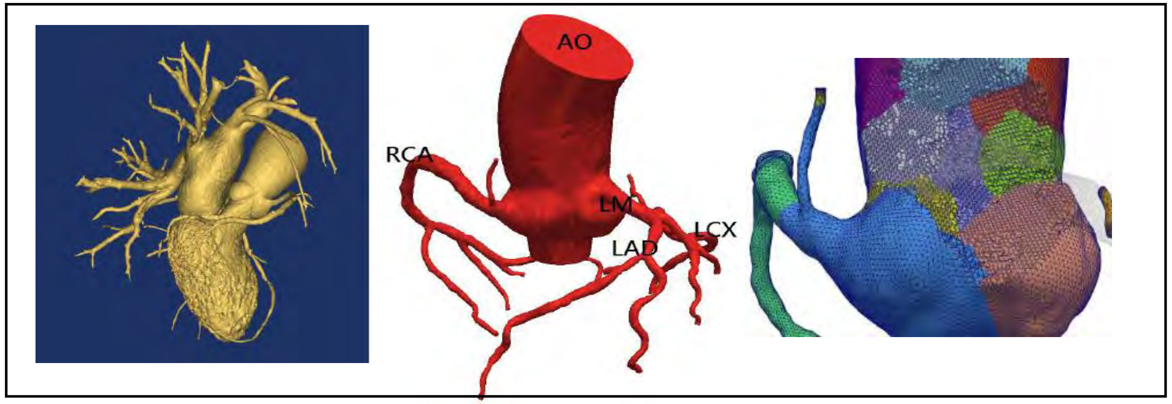
The cardiovascular system



http://slideplayer.com/slide/8422767/

- 3D geometries reconstructed from MRI or CTA images by Mimics®
- Volume meshes generated from ANSYS® and partitioned by ParMETIS

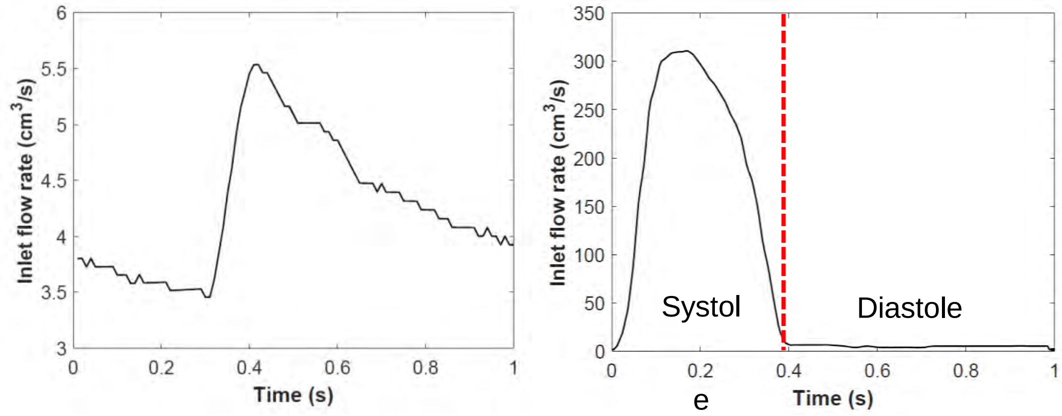




Computational Techniques

The 3D time-dependent incompressible Navier-Stokes equations:

$$
\begin{cases}
\rho\left(\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) - \nabla\cdot\boldsymbol{\sigma} = \mathbf{0}, & \text{in} \quad \Omega, \\
\boldsymbol{\sigma} = -p\mathbf{I} + \mu\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right), & \\
\nabla\cdot\mathbf{u} = 0, & \text{in} \quad \Omega, \\
\mathbf{u} = \mathbf{0}, & \text{on} \quad \Gamma_W, \\
\mathbf{u} = \mathbf{v}_I, & \text{on} \quad \Gamma_I, \\
\mathbf{u}|_{t=0} = \mathbf{u}_0, & \text{in} \quad \Omega.
\end{cases}
$$

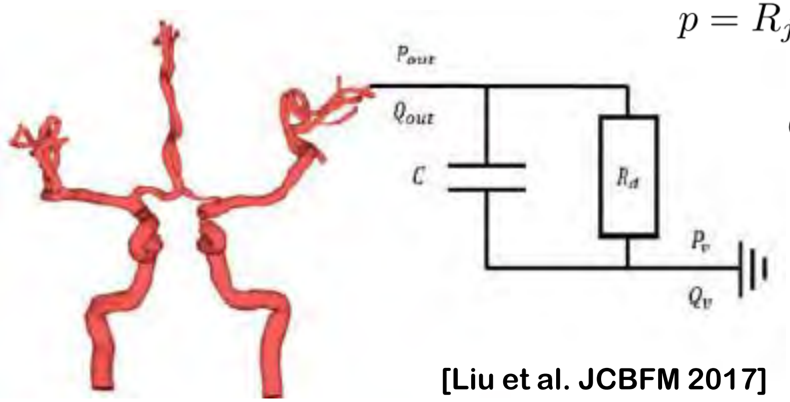


Inflow rates for the cerebral artery (left) and the coronary artery (right)

(i) The resistive condition for cerebral and coronary arteries

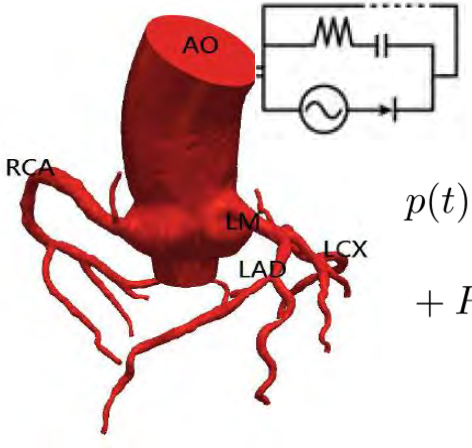
$$p = R_j Q_j, \quad j = 1, \dots, m$$

$$Q_j = \int_{\Gamma_{O_j}} \mathbf{u}\cdot\mathbf{n}_j \mathrm{d}\Gamma$$



[Liu et al. JCBFM 2017]

(ii) The impedance condition for aorta



$$p(t) = \left(P(0) - R_0 Q_0(0) - P_d(0)\right) e^{-t/\tau} + P_d(t) + R_0 Q_0 + \int_0^t \frac{e^{-(t-s)/\tau}}{C} Q_0(s) \mathrm{d}s$$

- **Temporal discretization:** Implicit backward Euler finite difference method
- **Spatial discretization :** P1-P1 stabilized finite element method based on unstructured mesh

Newton-Krylov-Schwarz algorithm

- **Newton:** Inexact Newton method with backtracking (INB)

$$\mathbf{x}_{k+1}^n = \mathbf{x}_k^n + \lambda_k s_k^n$$

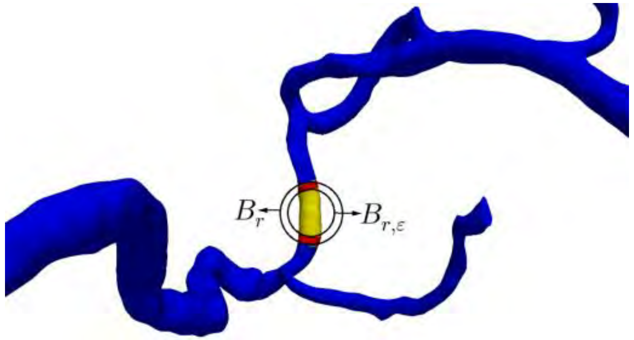
- **Krylov:** Krylov subspace method such as GMRES

$$J_k^n s_k^n = -\mathcal{F}\left(\mathbf{x}_k^n\right) \quad \text{with} \quad s_k^n = M_{RAS}^{-1} z$$

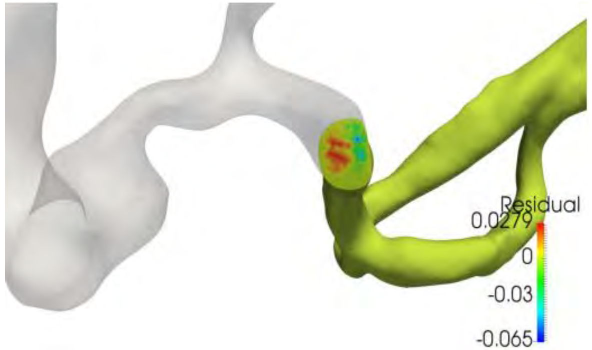
- **Schwarz:** Restricted additive Schwarz preconditioner

$$M_{RAS}^{-1} = \sum_{l=1}^{np} \left(R_l^0\right)^T \left(A_l\right)^{-1} R_l^\delta,$$

$$A_l = R_l^\delta A \left(R_l^\delta\right)^T.$$



The bad region B_r and its restricted part $B_{r,\varepsilon}$



Before the subspace correction

Nonlinear Elimination (NE) preconditioner

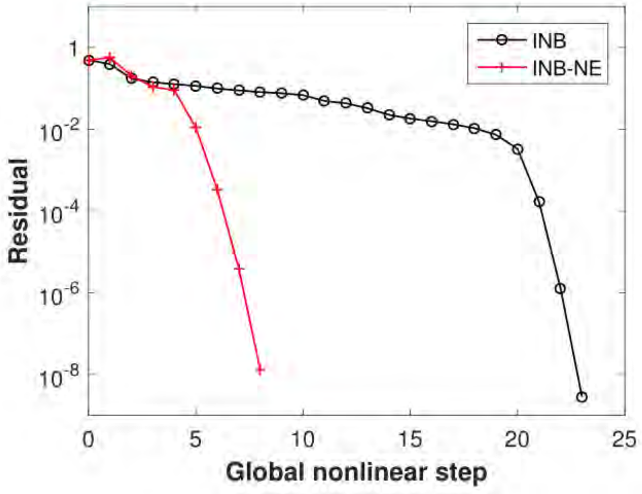
- Subspace correction by

$$\mathbf{y}_k = G\left(\mathcal{G}, \mathbf{x}_k^n\right) = \mathcal{R}_g^{k,\varepsilon}\left(\mathbf{x}_k^n\right) + \mathcal{R}_b^{k,\varepsilon}\left(\mathbf{x}^*\right)$$

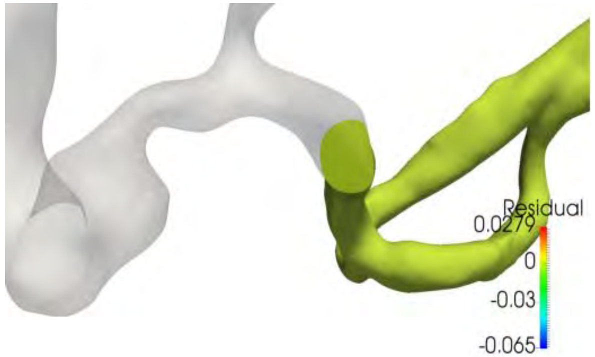
with $\mathbf{x}^*$ as the solution of

$$\mathcal{G}(\mathbf{x}) = \mathcal{R}_g^k\left(\mathbf{x} - \mathbf{x}_k^n\right) + \mathcal{R}_b^k\left(\mathcal{F}\left(\mathbf{x}\right)\right) = \mathbf{0}$$

- Update $\mathbf{x}_k^n = \mathbf{y}_k$ before solving the global nonlinear system



Nonlinear residual history

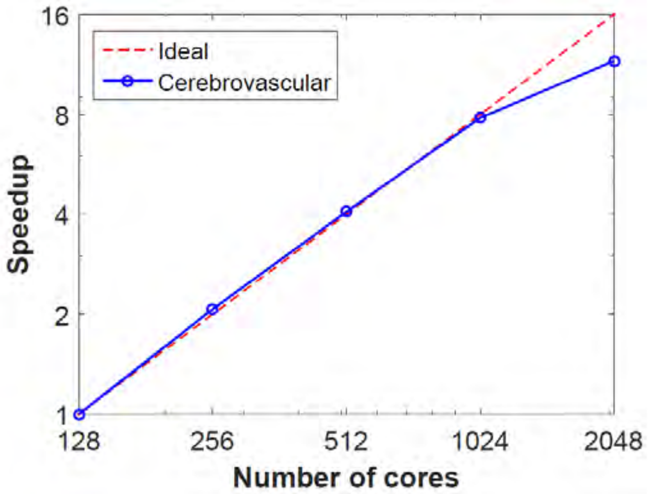


After the subspace correction

Numerical Results

Each node on Tianhe-2 supercomputer:

- ➢ **Two 12-core Intel Ivy Bridge Xeon CPUs**
- ➢ **24GB local memory**



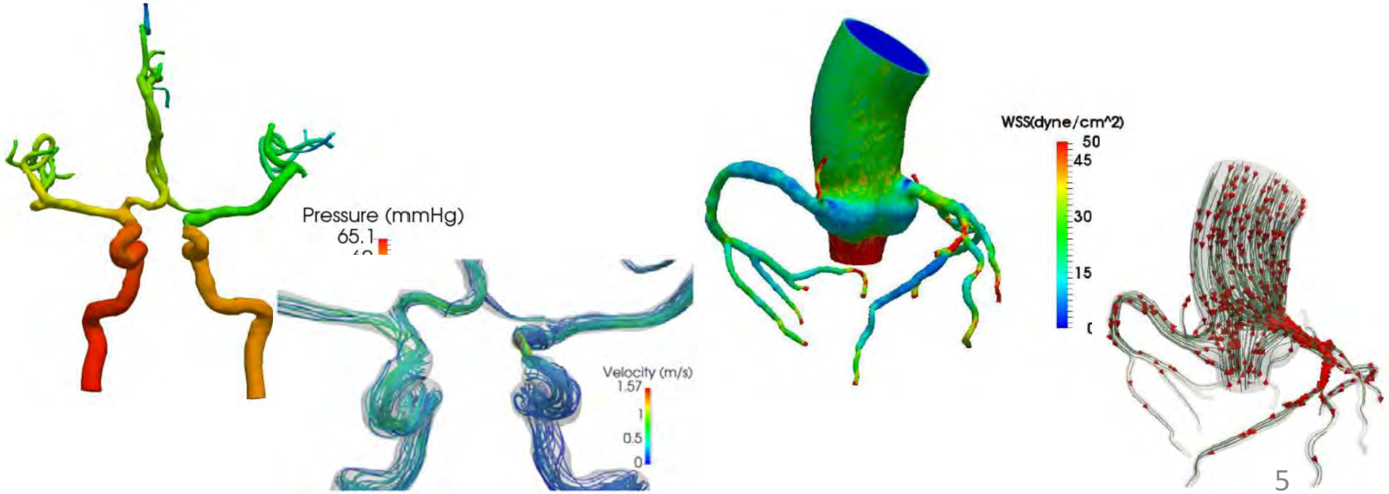
Strong scalability test for the cerebrovascular system with Dof=7,418,644

Comparison with different time step sizes

The classical INB method					
Δt (s)	0.005	0.01	0.02	0.04	0.06
NI_{global}	6.33	10.50	11.66	–	–
LI_{global}	522.16	592.75	605.17	–	–
Time_{total}(s)	253.95	458.04	537.44	–	–
The present INB-NE method					
Δt (s)	0.005	0.01	0.02	0.04	0.06
NI_{global}	5.50	5.67	7.00	8.17	8.83
LI_{global}	410.39	403.21	357.26	320.35	314.60
N_{ne}	1.33	1.67	1.33	1.50	1.50
NI_{ne}	2.63	3.20	2.87	2.56	3.00
LI_{ne}	16.48	17.19	10.74	11.74	15.44
Time_{ne}(s)	17.76	20.55	19.40	17.35	20.06
Time_{total}(s)	214.08	225.94	254.34	277.00	299.98

“--” means the case fails to converge.

Blood density: $\rho = 1.06\mathrm{g/cm^3}$, **blood viscosity:** $\mu = 0.035\mathrm{g/(cm{\cdot}s)}$



Thank you

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